Mestrado Profissional em Matemática em Rede
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Traduza o texto abaixo:

A satisfactory discussion of the main concepts of analysis (such as convergence, continuity, differentiation and integration) must be based on an accurately defined number concept. We shall not, however, enter into any discussion of the axioms that govern the arithmetic of the integers, but assume familiarity with the rational numbers (the numbers of the form m/n where m and n are integers and $n \neq 0$). The rational number system is inadequate for many purposes, both as a field and as an ordered set. For instance, there is no rational p such that $p^2 = 2$. This leads to the introduction of so-called "irrational numbers" which are often written as infinite decimal expansions and are considered to be "approximated" by the corresponding finite decimals. Thus the sequence $1, 1.4, 1.41, 1.414, 1.4142, ...$ "tends to $\sqrt{2}$". But unless the irrational number $\sqrt{2}$ has been clearly defined, the question must arise: Just what is it that this sequence "tends to"? This sort of question can be answered as soon as the so-called "real number system" is constructed.

Example: We now show that the equation

$$p^2 = 2 \quad (1)$$

is not satisfied by any rational p. If there were such a p, we could write $p = m/n$ where m and n are integers that are not both even. Let us assume this is done. Then (1) implies

$$m^2 = 2n^2. \quad (2)$$

This shows that m^2 is even. Hence m is even (if m were odd, m^2 would be odd), and so m^2 is divisible by 4. It follows that the right side of (2) is divisible by 4, so that n^2 is even, which implies that n is even. The assumption that (1) holds thus leads to the conclusion that both m and n are even, contrary to our choice of m and n. Hence (1) is impossible for rational p. We now examine this situation a little more closely. Let A be a set of all positive rationals p such that $p^2 < 2$ and let B consist of all positive rationals p such that $p^2 > 2$. We shall show that A contains no largest number and B contains no smallest. More explicitly, for every p in A we can find a rational q in A such that $p < q$, and for every p in B we can find a rational q in B such that $q < p$. To do this, we associate with each rational $p > 0$ the number

$$q = p - \frac{p^2 - 2}{p + 2} = \frac{2p + 2}{p + 2} \quad (3)$$

Then

$$q^2 - 2 = \frac{2(p^2 - 2)}{(p + 2)^2}. \quad (4)$$

If p is in A then $p^2 - 2 < 0$, (3) shows that $q > p$, and (4) shows that $q^2 < 2$. Thus q is in A. If p is in B then $p^2 - 2 > 0$, (3) shows that $0 < q < p$, and (4) shows that $q^2 > 2$. Thus q is in B. Remark: The purpose of the above discussion has been to show that the rational number system has certain gaps, in spite of the fact that between any two rationals there is another: If $r < s$ then $r < (r + s)/2 < s$. The real number system fills these gaps. This is the principal reason for the fundamental role which it plays in analysis.

Tradução:

A satisfactory discussion of the main concepts of analysis (such as convergence, continuity, differentiation and integration) must be based on an accurately defined number concept.Uma discussão satisfatória dos principais conceitos de análise (como convergência, continuidade, diferenciação e integração) deve ser baseada em um conceito numérico definido com precisão. We shall not, however, enter into any discussion of the axioms that govern the arithmetic of the integers, but assume familiarity with the rational numbers (the numbers of the form m/n where m and n are integers and $n \neq 0$).Não entraremos, no entanto, em nenhuma discussão sobre os axiomas que governam a aritmética dos inteiros, mas assumiremos familiaridade com os números racionais (os números da forma m/n onde m e n são inteiros e $n \neq 0$). The rational number system is inadequate for many purposes, both as a field and as an ordered set. O sistema de números racionais é inadequado para muitos propósitos, tanto como um corpo quanto como um conjunto ordenado. For instance, there is no rational p such that $p^2 = 2$.Por exemplo, não existe p racional tal que $p^2 = 2$. This leads to the introduction of so-called "irrational numbers"which are often written as infinite decimal expansions and are considered to be "approximated"by the corresponding finite decimals.Isso leva à introdução dos chamados "números irracionais"que são frequentemente escritos como expansões decimais infinitas e são considerados "aproximados"pelos decimais finitos correspondentes. Thus the sequence $1, 1.4, 1.41, 1.414, 1.4142, ...$ "tends to $\sqrt{2}$".Assim, a sequência $1, 1, 4, 1, 41, 1, 414, 1, 4142, ...$ "tende a $\sqrt{2}$". But unless the irrational number $\sqrt{2}$ has been clearly defined, the question must arise: Just what is it that this sequence "tends to"?Mas, a menos que o número irracional $\sqrt{2}$ tenha sido claramente definido, a questão deve surgir: Exatamente a que essa sequência "tende"? This sort of question can be answered as soon as the so-called "real number system"is constructed.Esse tipo de questão pode ser respondida assim que o chamado "sistema de números reais"for construído.

Example: We now show that the equation

$$p^2 = 2 \quad (1)$$

is not satisfied by any rational p. Exemplo: Agora mostramos que a equação

$$p^2 = 2 \quad (1)$$

não é satisfeita por nenhum p racional.If there were such a p, we could write $p = m/n$ where m and n are integers that are not both even.Se houvesse tal p, poderíamos escrever $p = m/n$ onde m e n são inteiros que não são ambos pares. Let us assume this is done. Then (1) implies

$$m^2 = 2n^2. \quad (2)$$

Vamos supor que isso seja feito. Então (1) implica

$$m^2 = 2n^2. \quad (2)$$

This shows that m^2 is even.Isso mostra que m^2 é par. Hence m is even (if m were odd, m^2 would be odd), and so m^2 is divisible by 4.Portanto, m é par (se m fosse ímpar, m^2 seria ímpar) e, portanto, m^2 é divisível por 4. It follows that the right side of (2) is divisible by 4, so that n^2 is even, which implies that n is even.Segue-se que o lado direito de (2) é divisível por 4, de modo que n^2 é par, o que implica que n é par. The assumption that (1) holds thus leads to the conclusion that both m and n are even, contrary to our choice of m and n.A suposição de que (1) é válida, portanto, leva à conclusão de que tanto m quanto n são pares, ao contrário da nossa escolha de m e n. Hence (1) is impossible for rational p.Portanto, (1) é impossível para o racional p.

We now examine this situation a little more closely.Agora examinaremos essa situação um pouco mais de perto. Let A be a set of all positive rationals p such that $p^2 < 2$ and let B consist of all positive rationals p such that $p^2 > 2$.Seja A um conjunto de todos os racionais positivos p tais que $p^2 < 2$ e seja B consistindo em todos os racionais positivos p tais que $p^2 > 2$. We shall show that A contains no largest number and B contains no smallest.Mostraremos que A não contém nenhuma cota superior e B não contém cota inferior. More explicitly, for every p in A we can find a rational q in A such that $p < q$, and for every p in B we can find a rational q in B such that $q < p$.Mais explicitamente, para cada p em A podemos encontrar um racional q em A tal que $p < q$, e para cada p em B podemos encontrar um racional q em B tal que $q < p$. To do this, we associate with each rational $p > 0$ the number

$$q = p - \frac{p^2 - 2}{p + 2} = \frac{2p + 2}{p + 2}. \quad (3)$$

Para fazer isso, associamos a cada racional $p > 0$ o número

$$q = p - \frac{p^2 - 2}{p + 2} = \frac{2p + 2}{p + 2}. \quad (3)$$

Then

$$q^2 - 2 = \frac{2(p^2 - 2)}{(p + 2)^2}. \quad (4)$$

Então

$$q^2 - 2 = \frac{2(p^2 - 2)}{(p + 2)^2}. \quad (4)$$

If p is in A then $p^2 - 2 < 0$, (3) shows that $q > p$, and (4) shows that $q^2 < 2$.Se p está em A então $p^2 - 2 < 0$, (3) mostra que $q > p$, e (4) mostra que $q^2 < 2$. Thus q is in A.Assim, q está em A. If p is in B then $p^2 - 2 > 0$, (3) shows that $0 < q < p$, and (4) shows that $q^2 > 2$.Se p está em B então $p^2 - 2 > 0$, (3) mostra que $0 < q < p$, e (4) mostra que $q^2 > 2$. Thus q is in B.Assim, q está em B.

Remark: The purpose of the above discussion has been to show that the rational number system has certain gaps, in spite of the fact that between any two rationals there is another: If $r < s$ then $r < (r + s)/2 < s$.Observação: O propósito da discussão acima foi mostrar que o sistema de números racionais tem certas lacunas, apesar do fato de que entre quaisquer dois racionais há outro: Se $r < s$ então $r < (r + s)/2 < s$. The real number system fills these gaps. This is the principal reason for the fundamental role which it plays in analysis.O sistema de números reais preenche essas lacunas. Esta é a principal razão para o papel fundamental que ele desempenha na análise.