

Mestrado Profissional em Matemática em Rede  
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Traduza os textos abaixo:

**Texto 1**

Let  $f$  be a map from  $A$  to  $B$ . We then write  $f : A \rightarrow B$ .  $f$  is said to be injective (or one-to-one into) if and only if for all  $a, b \in A$ ,  $f(a) = f(b)$  implies  $a = b$ .  $f$  is said to be surjective (or onto) if  $f(A) = B$ , that is, for all  $b \in B$ , there exists  $a \in A$  such that  $f(a) = b$ .  $f$  is said to be bijective (or one-to-one onto) if, and only if,  $f$  is both injective and surjective.

Let  $f : A \rightarrow B$  and  $g : B \rightarrow C$  be maps. The composition of  $f$  and  $g$  is the map  $h : A \rightarrow C$  such that  $h(a) = g(f(a))$  for all  $a \in A$ . The composition  $h$  is usually written as  $g \circ f$ . On a few occasions, we use the exponential notation  $a^f$  instead of  $f(a)$ , in which case the composition of  $f$  and  $g$  will be written as  $fg$ . It will be clear from the context which notation is used. Often it is convenient to draw a diagram in order to exhibit a collection of maps. The diagram is said to be commutative if and only if the composition of maps indicated in them depends only on the initial and the end points and not on the chosen path. In general, a map will not preserve the intersection or the union of subsets. However the following statement is true.

**Theorem 1.** Let  $f : A \rightarrow B$  be a map. Then  $f^{-1}(X \cup Y) = f^{-1}(X) \cup f^{-1}(Y)$  and  $f^{-1}(X \cap Y) = f^{-1}(X) \cap f^{-1}(Y)$  for any  $X, Y \subset B$ .

The proof of Theorem 1 is straightforward and will be left to the interested reader.

**Texto 2**

A vector space, also called a linear space, is a collection of objects called vectors, which may be added together and multiplied by numbers, called scalars. Scalars are often taken to be real numbers, but there are also vector spaces with scalar multiplication by complex numbers, rational numbers, or generally any field. The operations of vector addition and scalar multiplication must satisfy certain requirements, called axioms. For specifying that the scalars are real or complex numbers, the terms real vector space and complex vector space are often used.

If two vectors point in different directions, then the two vectors are said to be linearly independent. If vectors  $\vec{u}$  and  $\vec{w}$  point in the same direction, then you can multiply vector  $\vec{u}$  by a constant scalar value, and get vector  $\vec{w}$ , and vice versa to get  $\vec{u}$  from  $\vec{w}$ . If the two vectors point in different directions, then this is not possible to make one out of the other because multiplying a vector by a scalar will never change the direction of the vector, it will only change the magnitude.

This concept can be generalized to families of more than two vectors. Three vectors are said to be linearly independent if there is no way to construct one of them by combining scaled versions of the other two. The same definition applies to families of four or more vectors by applying the same rules. We can now write the formal definition of linear independence.

**Definition 1.** A family of vectors is linearly independent if no one of the vectors can be created by any linear combination of the other vectors in the family.

If a family of vectors are not linearly independent then the vectors are said to be linearly dependent. We can clearly rewrite the above definition into a new and equivalent definition of linear independence.

**Definition 2.** A set of vectors  $\{\vec{u}_1, \vec{u}_2, \vec{u}_3, \dots, \vec{u}_n\}$  is said to be linearly independent if and only if the equality

$$\alpha_1 \vec{u}_1 + \alpha_2 \vec{u}_2 + \alpha_3 \vec{u}_3 + \dots + \alpha_n \vec{u}_n = \vec{0},$$

implies

$$\alpha_1 = \alpha_2 = \alpha_3 = \dots = \alpha_n = 0.$$

## Tradução Texto 1

Let  $f$  be a map from  $A$  to  $B$ . Seja  $f$  uma aplicação de  $A$  para  $B$ . We then write  $f : A \rightarrow B$ . Então escrevemos  $f : A \rightarrow B$ .  $f$  is said to be injective (or one-to-one into) if and only if for all  $a, b \in A$ ,  $f(a) = f(b)$  implies  $a = b$ .  $f$  é dita ser injetiva (ou um-para-um em) se e somente se para todo  $a, b \in A$ ,  $f(a) = f(b)$  implica  $a = b$ .  $f$  is said to be surjective (or onto) if  $f(A) = B$ , that is, for all  $b \in B$ , there exists  $a \in A$  such that  $f(a) = b$ .  $f$  é dita ser sobrejetiva (ou sobre) se  $f(A) = B$ , isto é, para todo  $b \in B$ , existe  $a \in A$  tal que  $f(a) = b$ .  $f$  is said to be bijective (or one-to-one onto) if, and only if,  $f$  is both injective and surjective.  $f$  é dita ser bijetiva (ou um-para-um sobre) se, e somente se,  $f$  é tanto injetiva quanto sobrejetiva.

Let  $f : A \rightarrow B$  and  $g : B \rightarrow C$  be maps. Sejam  $f : A \rightarrow B$  e  $g : B \rightarrow C$  aplicações. The composition of  $f$  and  $g$  is the map  $h : A \rightarrow C$  such that  $h(a) = g(f(a))$  for all  $a \in A$ . A composição de  $f$  e  $g$  é a aplicação  $h : A \rightarrow C$  tal que  $h(a) = g(f(a))$  para todo  $a \in A$ . The composition  $h$  is usually written as  $g \circ f$ . A composição  $h$  é geralmente escrita como  $g \circ f$ . On a few occasions, we use the "exponential" notation  $a^f$  instead of  $f(a)$ , in which case the composition of  $f$  and  $g$  will be written as  $fg$ . Em algumas ocasiões, usamos a notação "exponencial"  $a^f$  em vez de  $f(a)$ , caso em que a composição de  $f$  e  $g$  será escrita como  $fg$ . It will be clear from the context which notation is use. Ficará claro pelo contexto qual notação é usada. Often it is convenient to draw a diagram in order to exhibit a collection of maps. Frequentemente é conveniente desenhar um diagrama para exibir uma coleção de aplicações. The diagram is said to be commutative if and only if the composition of maps indicated in them depends only on the initial and the end points and not on the chosen path. O diagrama é dito comutativo se e somente se a composição de aplicações indicadas neles depende apenas dos pontos inicial e final e não do caminho escolhido. In general, a map will not preserve the intersection or the union of subsets. However the following statement is true. Em geral, uma aplicação não preservará a interseção ou a união de subconjuntos. No entanto, a seguinte afirmação é verdadeira.

**Theorem 1.** Let  $f : A \rightarrow B$  be a map. **Teorema 1.** Seja  $f : A \rightarrow B$  uma aplicação. Then  $f^{-1}(X \cup Y) = f^{-1}(X) \cup f^{-1}(Y)$  and  $f^{-1}(X \cap Y) = f^{-1}(X) \cap f^{-1}(Y)$  for any  $X, Y \subset B$ . Então  $f^{-1}(X \cup Y) = f^{-1}(X) \cup f^{-1}(Y)$  e  $f^{-1}(X \cap Y) = f^{-1}(X) \cap f^{-1}(Y)$  para qualquer  $X, Y \subset B$ .

The proof of Theorem 1 is straightforward and will be left to the interested reader. A prova do Teorema 1 é direta e será deixada para o leitor interessado.

## Tradução Texto 2

A vector space, also called a linear space, is a collection of objects called vectors, which may be added together and multiplied by numbers, called scalars. Um espaço vetorial, também chamado de espaço linear, é uma coleção de objetos chamados vetores, que podem ser somados e multiplicados por números, chamados escalares. Scalars are often taken to be real numbers, but there are also vector spaces with scalar multiplication by complex numbers, rational numbers, or generally any field. Os escalares são frequentemente considerados números reais, mas também há espaços vetoriais com multiplicação escalar por números complexos, números racionais ou geralmente qualquer corpo. The operations of vector addition and scalar multiplication must satisfy certain requirements, called axioms. As operações de adição vetorial e multiplicação escalar devem satisfazer certos requisitos, chamados axiomas. For specifying that the scalars are real or complex numbers, the terms real vector space and complex vector space are often used. Para especificar que os escalares são números reais ou complexos, os termos espaço vetorial real e espaço vetorial complexo são frequentemente usados.

If two vectors point in different directions, then the two vectors are said to be linearly indepen-

dent. Se dois vetores apontam em direções diferentes, então os dois vetores são considerados linearmente independentes. If vectors  $\vec{u}$  and  $\vec{w}$  point in the same direction, then you can multiply vector  $\vec{u}$  by a constant scalar value, and get vector  $\vec{w}$ , and vice versa to get  $\vec{u}$  from  $\vec{w}$ . Se os vetores  $\vec{u}$  e  $\vec{w}$  apontam na mesma direção, então você pode multiplicar o vetor  $\vec{u}$  por um valor escalar constante, e obter o vetor  $\vec{w}$ , e vice-versa para obter  $\vec{u}$  de  $\vec{w}$ . If the two vectors point in different directions, then this is not possible to make one out of the other because multiplying a vector by a scalar will never change the direction of the vector, it will only change the magnitude. Se os dois vetores apontam em direções diferentes, então não é possível fazer um do outro porque multiplicar um vetor por um escalar nunca mudará a direção do vetor, apenas mudará a magnitude.

This concept can be generalized to families of more than two vectors. Este conceito pode ser generalizado para famílias de mais de dois vetores. Three vectors are said to be linearly independent if there is no way to construct one of them by combining scaled versions of the other two. Três vetores são considerados linearmente independentes se não houver maneira de construir um deles combinando versões escalonadas dos outros dois. The same definition applies to families of four or more vectors by applying the same rules. A mesma definição se aplica a famílias de quatro ou mais vetores aplicando as mesmas regras. We can now write the formal definition of linear independence. Agora podemos escrever a definição formal de independência linear.

**Definition 1.** A family of vectors is linearly independent if no one of the vectors can be created by any linear combination of the other vectors in the family. **Definição 1.** Uma família de vetores é linearmente independente se nenhum dos vetores pode ser criado por qualquer combinação linear dos outros vetores na família.

If a family of vectors are not linearly independent then the vectors are said to be linearly dependent. Se uma família de vetores não for linearmente independente, então os vetores são considerados linearmente dependentes. We can clearly rewrite the above definition into a new and equivalent definition of linear independence. Podemos reescrever claramente a definição acima em uma nova e equivalente definição de independência linear.

**Definition 2.** A set of vectors  $\{\vec{u}_1, \vec{u}_2, \vec{u}_3, \dots, \vec{u}_n\}$  is said to be linearly independent if and only if the equality

$$\alpha_1 \vec{u}_1 + \alpha_2 \vec{u}_2 + \alpha_3 \vec{u}_3 + \dots + \alpha_n \vec{u}_n = \vec{0},$$

implies

$$\alpha_1 = \alpha_2 = \alpha_3 = \dots = \alpha_n = 0.$$

**Definição 2.** Um conjunto de vetores  $\{\vec{u}_1, \vec{u}_2, \vec{u}_3, \dots, \vec{u}_n\}$  é dito ser linearmente independente se e somente se a igualdade

$$\alpha_1 \vec{u}_1 + \alpha_2 \vec{u}_2 + \alpha_3 \vec{u}_3 + \dots + \alpha_n \vec{u}_n = \vec{0},$$

implica

$$\alpha_1 = \alpha_2 = \alpha_3 = \dots = \alpha_n = 0.$$